The Capability of Error Correction for Burst-noise Channels Using Error Estimating Code

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Overview

Error Estimating Code

1. Error estimating code (EEC) [Chen et.al 2010] is used to estimate the bit-error-rate (BER) of a packet using a small number of random parity bits.
2. The EEC parity bits are divided into $l = \lfloor \log n \rfloor$ levels, and each level has s parity bits.
3. On the sender/encoder side, to generate a parity bit on level $1 \le i \le l$, we randomly and uniformly choose $2^i - 1$ data bits for parity check.
4. On the receiver/decoder side, for a given EEC bit e_j, we know the data bits checked by e_j by using the same random seed with the sender.

Error Estimating Code

The following is an illustrated EEC encoding:

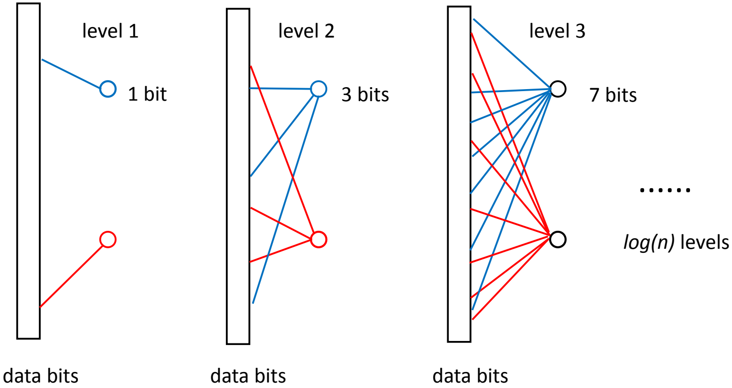


Figure: EEC encoding when $s = 2$

Error Estimating Code

Error estimating code has the following nice properties:

- high precision,
- low redundancy,
- low computational complexity.

Find applications in several scenarios like

- BER-based WiFi rate adaptation,
- packet retransmission,
- BER-aware routing.

Our Task

But can we also use EEC for error correction?

1. The capability of EEC for error correction has not been investigated before.
2. If we can use EEC for error correction to some extent, that will reduce the retransmission and improve performance.

Challenges

We face several challenges:

1. **Low redundancy.** If the τ erroneous bits are completely randomly distributed in the received packet, there will be $\binom{n}{\tau}$ equally possible index sets.
2. **Randomized nature.** EEC does not seem to have very strong structure as typical error correction codes like Reed-Solomon code.

Error Correction Capability

Theorem (error correction capability, simplified version)

For any constant $0 < \epsilon < 1$, EEC can identify the error bits with probability at least $1 - \epsilon$ as long as $s = \Omega_\epsilon(\tau)$, where τ is the number of error bits.

1. The probability is over the randomness in EEC encoding.
2. This is a general result, i.e. we do not make any assumptions on the distribution of error bits.

Burst-noise Channel

In many communication channels, erroneous bits are not randomly distributed, but spatially concentrated. We call such channels *burst-noise channel*.

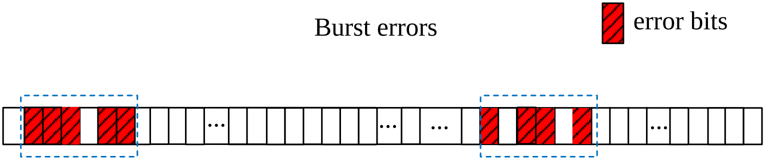


Figure: Packet with burst errors

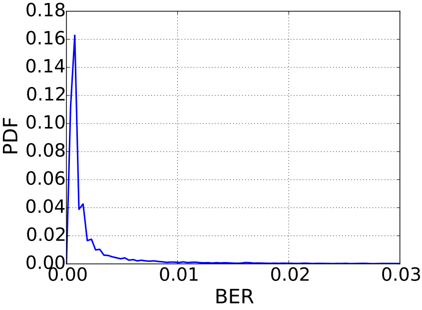
Burst-noise Channel

Definition (μ-Guard Burst)

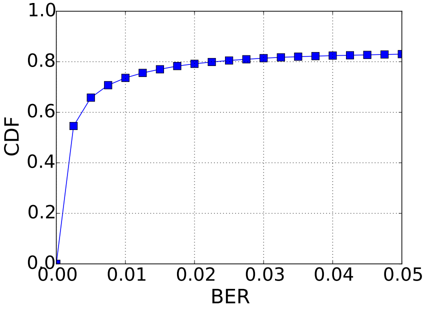
A μ-guard burst error is a sequence of corrupted data bits in a packet such that (1) the first and the last bits are erroneous, and (2) the maximum length of continuous correct bits within the burst is less than μ, where μ is an integer known as the *guard band* of the burst error.

Experimental Evaluation of Burst-noise Channel

WAVES trace collected by Wireless and Video (WAVES) Lab from Michigan State University.



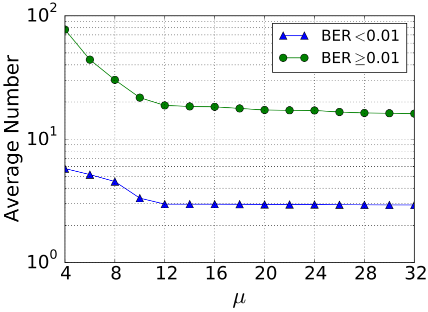
PDF of bit error rate.



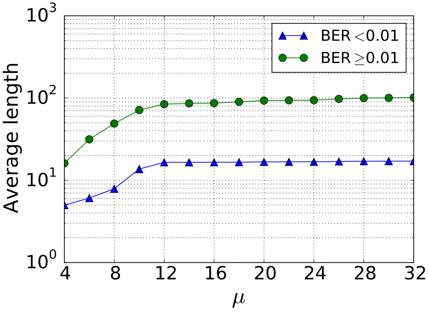
CDF of bit error rate.

Experimental Evaluation of Burst-noise Channel, continued

Varying the guard band μ, we analyze the average number and length of burst errors in the corrupted packets.



Average number of bursts.



Average length of bursts.

Overview of the Error Correction Scheme

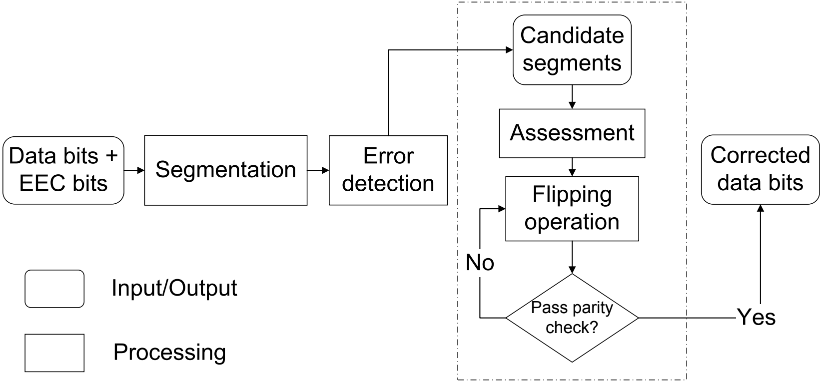


Figure: The error correction scheme.

Segmentation and Error Detection

1. Due to the burst-error property, only a few segments contain burst errors, and other segments are error-free.
2. For each segment $\vec{D}'_i$, we can define an indicator function $g(\cdot)$ to indicate the possibility that a segment contains burst-errors as

$$g(\vec{D}'_i) = \min_{\mathcal{A} \subseteq \mathcal{I}(\vec{D}'_i)} f(\vec{D}, \vec{D}' \bigtriangledown \mathcal{A}),$$

where $\mathcal{A}$ is a set of data bits in $\vec{D}'_i$ and $f(\cdot)$ indicates the effect of flipping $\mathcal{A}$.

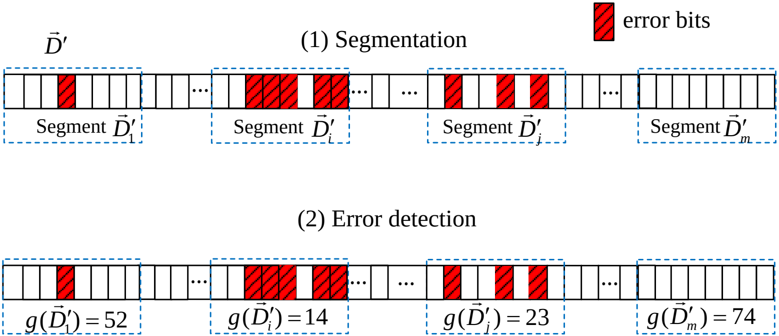


Figure: Segmentation and error detection.

Detecting Burst Segments

Let $\mathcal{D}_E$ be the set of erroneous segments, then we can prove the following theorem with repeated use of union bound and Chebyshev's inequality:

Theorem (Separability of Burst Segments)

Let A be the event that $g(\vec{D'_\alpha}) < g(\vec{D'_\beta})$ for any $\vec{D'_\alpha} \in \mathcal{D}_E$ and $\vec{D'_\beta} \notin \mathcal{D}_E$. $\forall \epsilon \in (0, 1)$, the following holds:

$$\Pr[A] > 1 - \epsilon, \tag{1}$$

when s is sufficiently large.

$g(\cdot)$ indicates erroneous segments

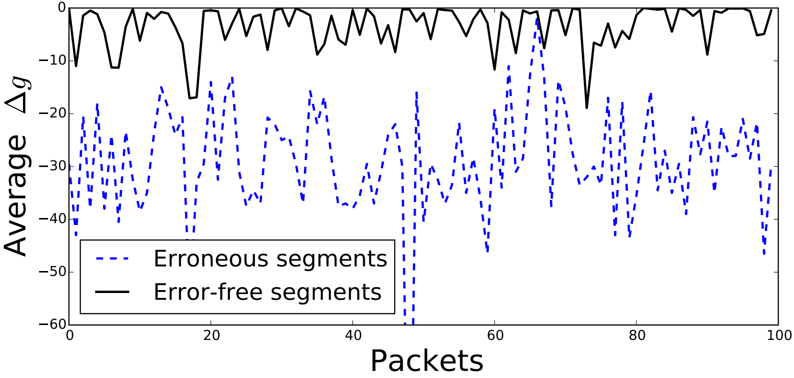


Figure: Average $g(\cdot)$ values for 100 random packets.

Candidate Segments

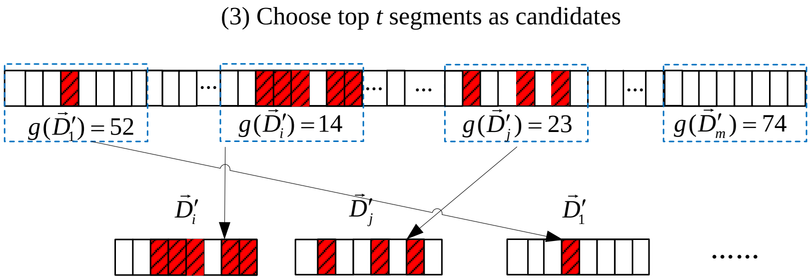


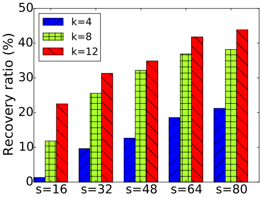
Figure: Candidate segments

Assessment and Flipping Procedure

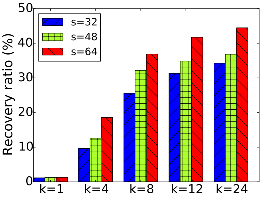
We propose a heuristic error correction algorithm based on erroneous assessment and flipping operation as follows.

- **Step 1: Initialization.** Let $\theta = (1 \pm \epsilon)\hat{p}n$; let the index set of erroneous bits $\mathcal{E} = \emptyset$.
- **Step 2: Assessment.** For each bits $b_i \in \mathcal{I}$, calculate the value of $g(b_i)$.
- **Step 3: Iteration.**
  (1) Select a set of bits $\mathcal{E}_\theta$ with size θ from $\mathcal{I}$ which have the smallest $g(\cdot)$ values.
  (2) Enumerate all combination of $(1 \pm \epsilon)\hat{p}n$ bits in $\mathcal{E}_\theta$. If the index set $\mathcal{E}'$ of a combination satisfies $f(\vec{D}, \vec{D}' \bigtriangledown \mathcal{E}') = 0$, let $\mathcal{E} = \mathcal{E}'$. Go to Step 4.
  (3) Let $\theta = \theta + (1 \pm \epsilon)\hat{p}n$. If $\theta \geq |\mathcal{I}|$ go to Step 4; else goto Step 3.
- **Step 4: Output.** If $\mathcal{E} \neq \emptyset$, output $\vec{D}' \bigtriangledown \mathcal{E}$; else fail to recover.

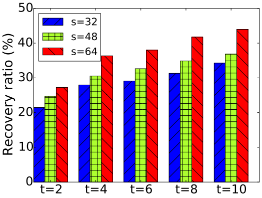
Our experiments show that we can correct over 40% corrupted packets.



Recovery ratio vs s.



Recovery ratio vs k.



Recovery ratio vs t.

Conclusion

1. With EEC we can identify all error bits with high probability if the BER is relatively low.
2. Characterize the burst-noise channels.
   - Burst-error phenomenon widely exists and can be used for error correction.
3. Design an EEC-based error correction algorithm for burst-noise channels, and give
   - theoretical analysis,
   - an efficient error correction scheme.
4. Performance evaluation, over 40% erroneous packets are corrected.

The End