

# Coverage for Target Localization in Wireless Sensor Networks

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# Overview

- Introduction
  - Sufficient condition for full coverage for localization applications
  - Estimating the necessary network density for localization
    - Disk coverage model
    - Sector coverage model
  - Conclusion
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# Coverage problems in sensor networks

- Coverage problem: Find a way to deploy or schedule sensors so that the *information loss* of the sensor network is below a given bound
- Information loss: the differences between the sensed data and the physical world
  - Depends on applications, can be measured as *target detection rate*, *localization error* or *data sampling rate* (in time or space)
- Conventional coverage models
  - Disk coverage model
  - Exposure model

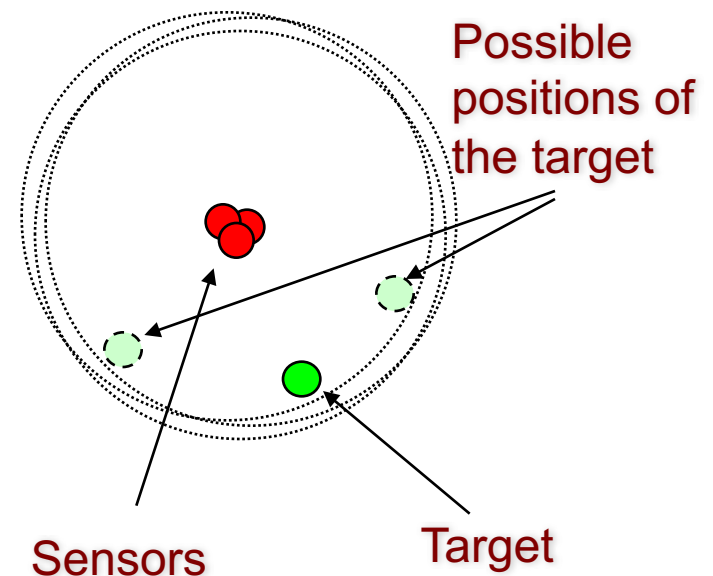
# Insufficiency of the disk coverage model

- Disk model

- Sensing area is a disk around sensors
- A point is deemed to be covered when it is within the sensing area

- Insufficient for localization

- Detected target can be localized with certain accuracy
- A  $k$ -covered target still may not be accurately localized



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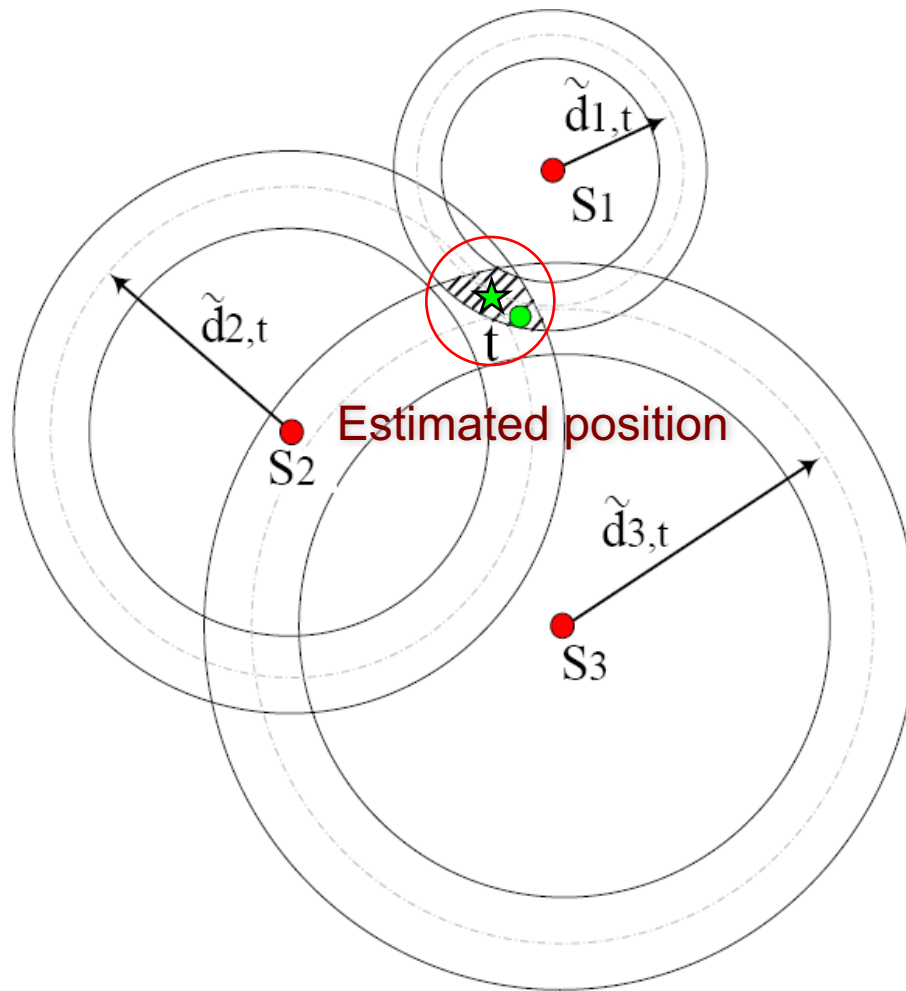
# Problems addressed in this paper

- How to guarantee a localization error bound in a sensor network?
  - The relationship between sensor density and the localization error in a random network
  - Methods for estimating sensor density when given the localization error requirement. Can we apply disk model in this case?
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# Localization model used in this paper

- The locations for sensors are known
- Sensors can measure its distance to the target
- When the estimated distance is  $d_{i,t}^o$ , then the true distance between sensor  $i$  and target  $t$  is within  $[d_{i,t}^o - e, d_{i,t}^o + e]$  with high probability, where  $e$  is the error bound when target is within detection range

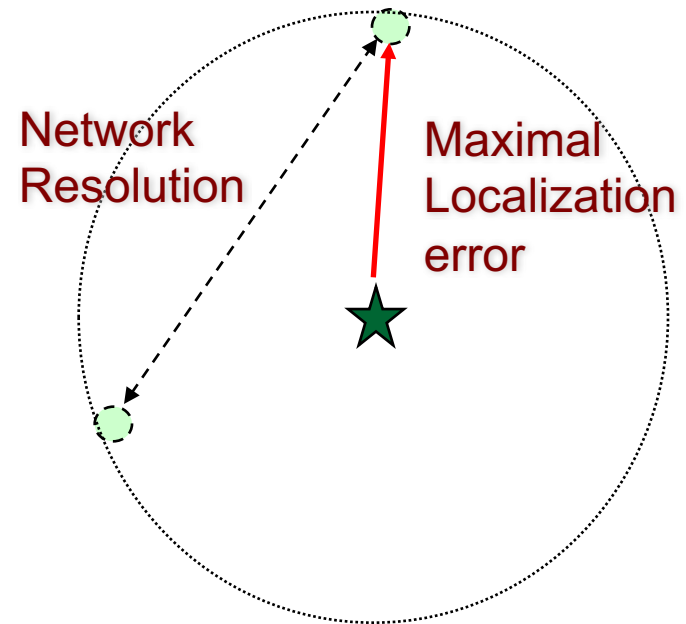
# Localizing through multiple sensors



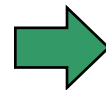
- Each sensor can localize the target within a ring of  $2e$
- Use the center of the intersection area as the estimated target location
- Localization error depends on the size of intersection area

# Network resolution

- The network can distinguish any pair of two point, which are departed more than the *network resolution*, through the distance measurements
- Relationship between network resolution and localization error

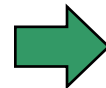


Network resolution  $< \sqrt{3}\varepsilon$



Localization error  $< \varepsilon$

Network resolution  $> 2\varepsilon$



Localization error  $> \varepsilon$



# Sufficient condition for localization coverage

*Theorem 1:*

If there is no  $(r, \frac{2}{3})$  sector shaped void in the network

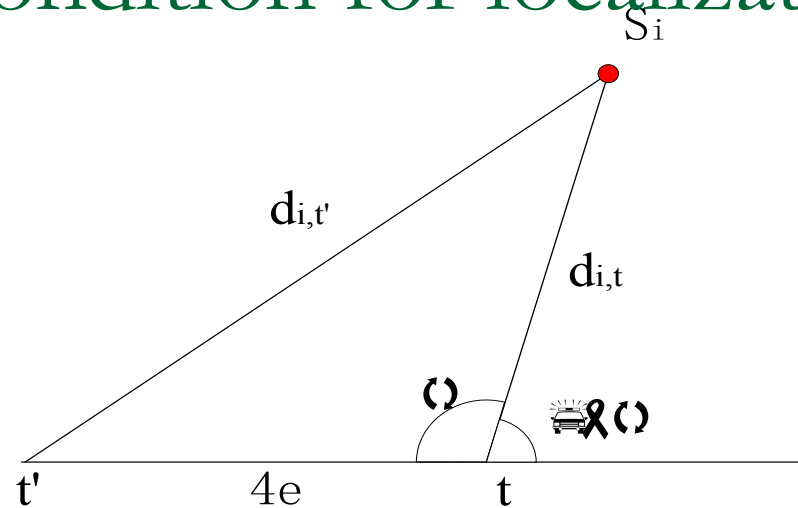


Every pair of points can be distinguished by at least one sensor when they are separated by more than  $4e$ . Thus the network resolution is smaller than  $4e$



The localization error over the network is bounded by  $\frac{4\sqrt{3}}{3}e$

# Sufficient condition for localization coverage

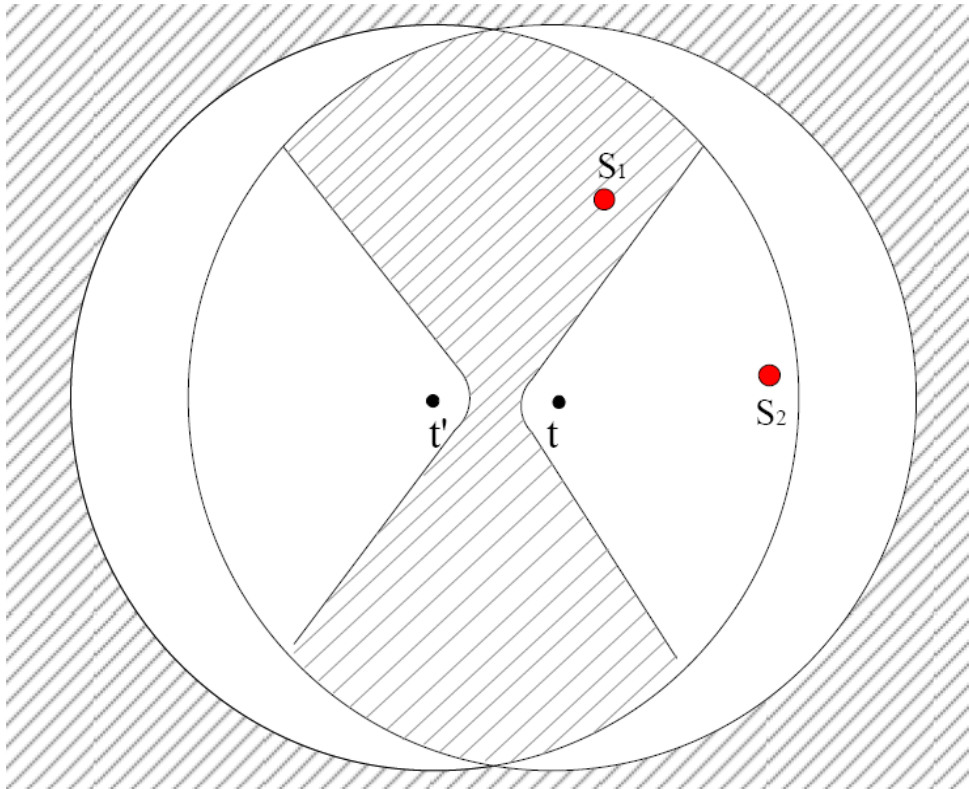


Suppose sensor  $S_i$  can not distinguish  $t$  and  $t'$

$$d_{i,t} = d_{i,t'} + 2e \quad \Rightarrow \quad \cos \theta = \frac{d_{i,t}^2 + (4e)^2 - d_{i,t'}^2}{2d_{i,t} \cdot 4e} = \frac{1}{2}$$

Then  $\theta$  will be larger than  $\pi/3$

# Sufficient condition for localization coverage



- Sensor  $S_1$  can not distinguish point  $t$  and  $t'$ , but sensor  $S_2$  can
- If there are any sensor in the white area, as a  $2/3$  sector, the two point can be distinguished

# Link back to disk coverage

## Disk coverage model

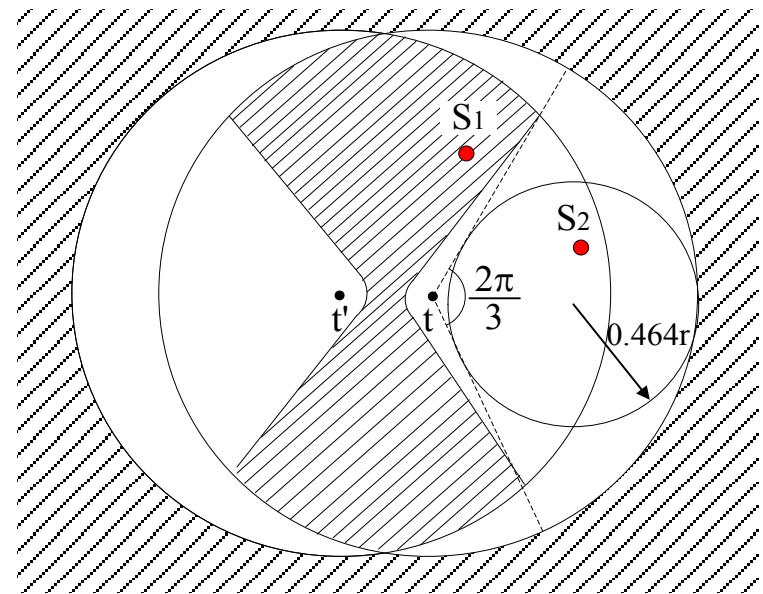
- Ensure at least one sensor in every disk with radius of  $0.464r$  ( $r$  is the detection range)
- In other words, need more than **4** times sensor for coverage than detection

## Benefits

- Disk model is well studied
- Can use simple algorithms to check coverage

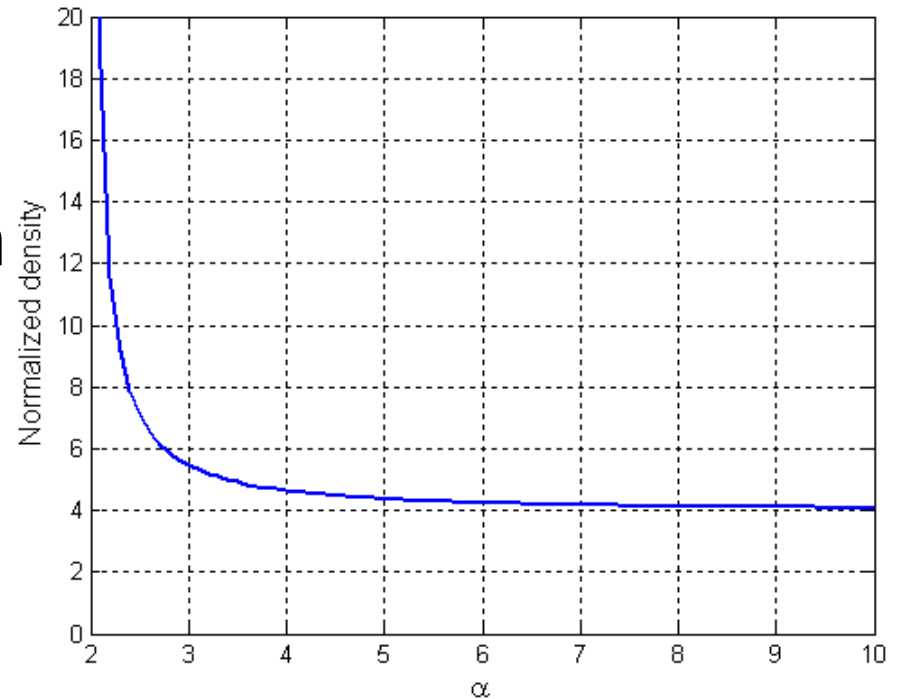
## Drawbacks

- overestimates the density



# Why select network resolution as $4e$

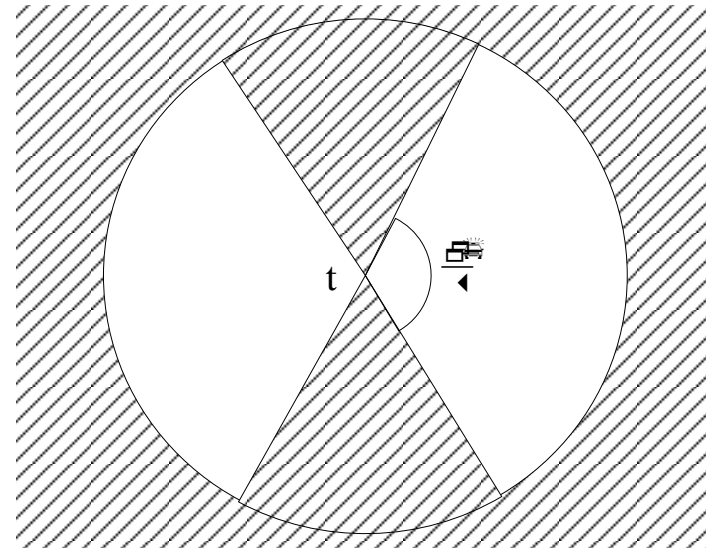
- Network resolution can be selected as  $\alpha e$
- We need extremely high density to guarantee a small  $\alpha$
- Even if we select  $\alpha$  as much larger than 4, we still need nearly same density as  $\alpha=4$



Relationship between density and network resolution

# Sector coverage

- Disk model is too strict and overestimates the density
- Sector coverage:
  - approximate the void as two opposite sectors
  - meet the coverage condition by ensuring nonexistence of such sector shaped void

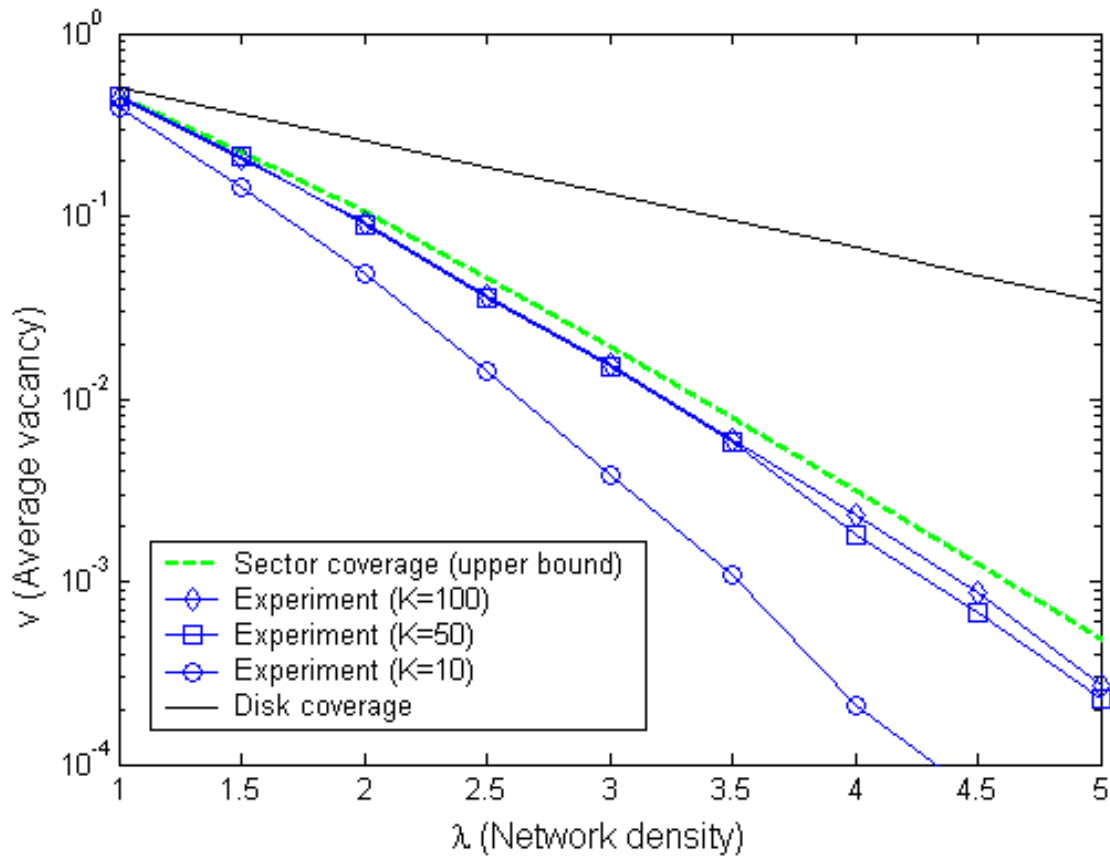


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# Estimating density through sector coverage

- Assume sensors are uniformly distributed (Poisson Point Process)
  - Given the density of sensors, estimate the probability that there are two opposite sector void around one point
  - Provides more accurate density estimations, require 2 times less sensor than disk model
  - More complex than disk model
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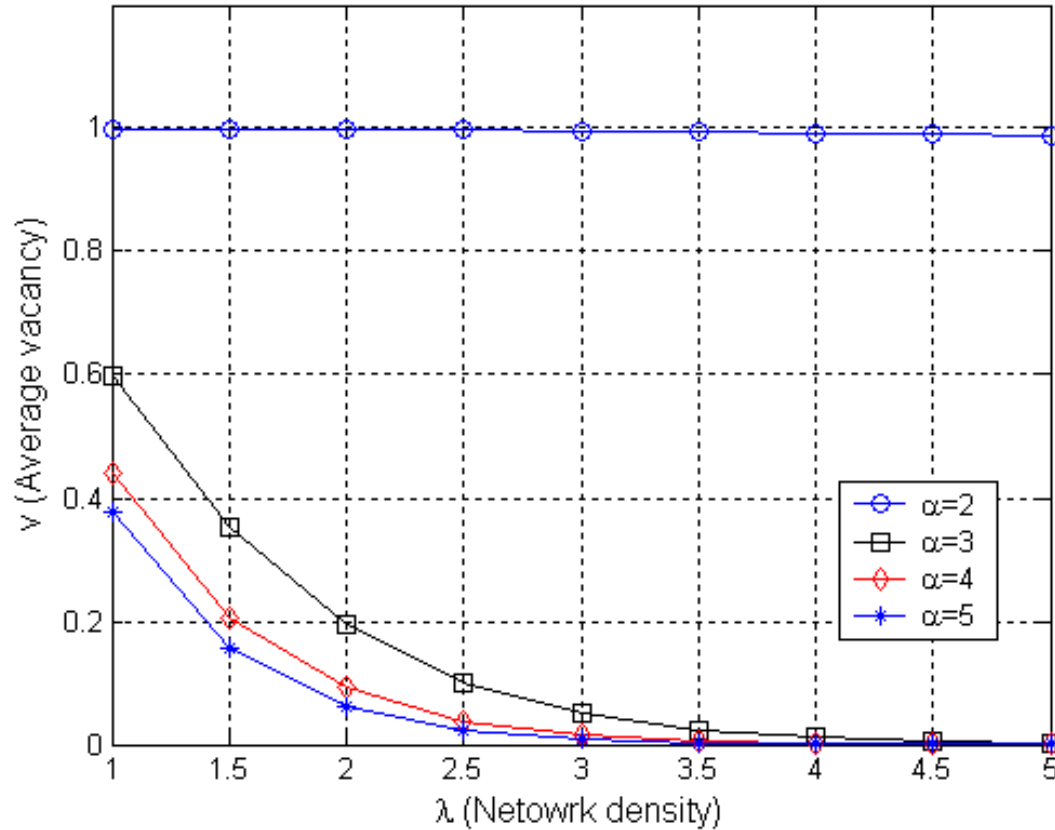
# Simulation results



Estimation of vacancy through disk and sector coverage



# Simulation results



Vacancy size with different network resolution requirements

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# Conclusion

- Disk coverage is insufficient for some applications
  - We show the sufficient condition for coverage of localization
  - Two methods for estimating the necessary density for localization
    - Disk coverage:* Require nearly 4 time more sensors than detection, low computational complexity
    - Sector coverage:* More accurate estimation, use 2 times less sensors, require complex algorithms
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Thank you!

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Supplementary slides

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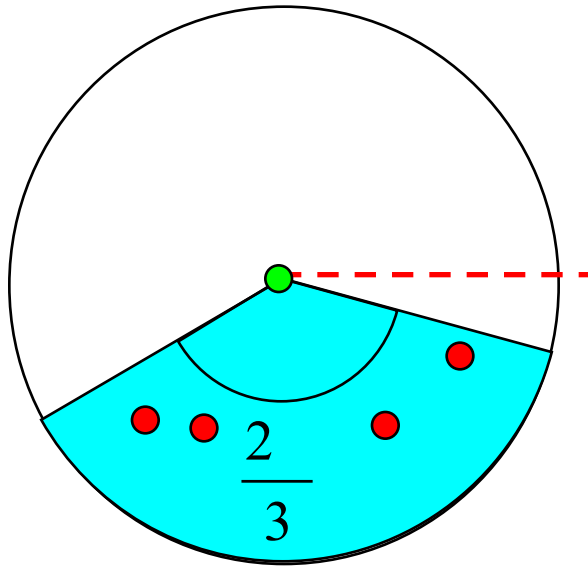
# Estimate density through sector coverage

- The probability for there are  $k$  sensors within the detection range

$$P_k = e^{-r^2} \frac{(r^2)^k}{k!}$$

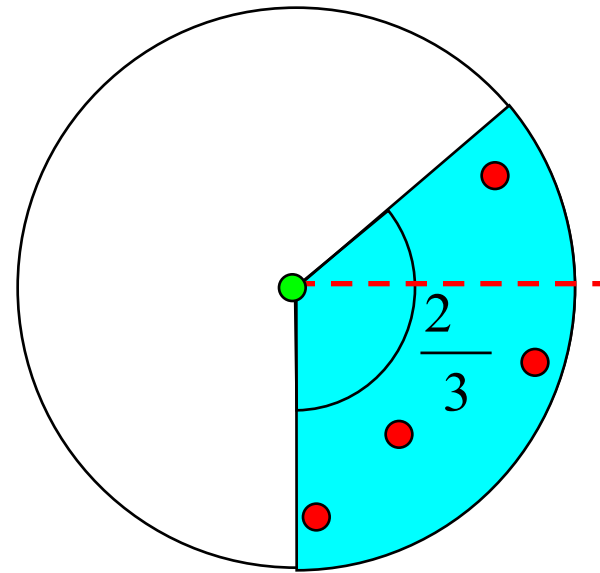
- The probability that there are two  $2\pi/3$  sector voids given there are  $k$  sensors around
  - $k=0, 1$ : probability=1
  - $k=2$ : probability=2/3
  - $k>2$ : probability=exist one  $4\pi/3$  sector void

# Probability of $4\pi/3$ sector void when $k$ -covered



$Q_k$ : All the  $k$  sensors in a  $2\pi/3$  sector not crossing the zero angle

$$Q_k = k \frac{1}{3} \binom{k-1}{k-1} \frac{1}{3}^k$$



$Q'_k$ : All the  $k$  sensors in a  $2\pi/3$  sector crossing the zero angle

$$Q'_k = \frac{1}{2} Q_k$$

# Estimate density through sector coverage

- Overall the probability that a point is uncovered in sector model is bounded by

$$P_0 + P_1 + \frac{2P_2}{3} + \sum_{k=3} Q_k P_K \quad P_{\text{sector}} \quad P_0 + P_1 + \frac{2P_2}{3} + \frac{3}{2} \sum_{k=3} Q_k P_K$$

- The probability that a point is uncovered in disk model

$$P_{\text{disk}} = e^{-(0.464r)^2}$$