Control of atomic spin squeezing via quantum coherence

Xuping Shao,1,2 Yang Ling,1 Xihua Yang,1,* and Min Xiao3,4
1Department of Physics, Shanghai University, Shanghai 200444, China
2School of Science, Nantong University, Nantong 226019, China
3National Laboratory of Solid State Microstructures and School of Physics, Nanjing University, Nanjing 210093, China
4Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA

(Received 9 August 2015; published 13 June 2016)

We propose a scheme to generate and control atomic spin squeezing via atomic coherence induced by the strong coupling and probe fields in the Λ-type electromagnetically-induced-transparency configuration in an atomic ensemble. Manipulation of squeezing of the two components in the plane orthogonal to the mean atomic spin direction and generation of nearly perfect squeezing in either component can be achieved by varying the relative intensities of the coupling and probe fields. This method provides a flexible and convenient way to create and control atomic spin squeezing, which may find potential applications in high-precision atomic-physics measurements, quantum coherent control, and quantum information processing.

DOI: 10.1103/PhysRevA.93.063825

I. INTRODUCTION

Strong light-matter interactions may result in many peculiar effects, such as coherent population trapping [1], electromagnetically induced transparency (EIT) [2], rapid adiabatic population transfer [3,4], and quantum correlation and entanglement [5–15]. The underlying physical mechanism of these special effects is quantum coherence. Much attention has been paid to the generation of the squeezed or entangled state of atomic spin during the past two decades due to its potential applications in high-precision atomic-physics measurements and quantum information processing [16–20]. Generally, the realization of spin squeezing in atomic systems can be divided into four categories [18,19]: (1) quantum nondemolition (QND) measurement of spin [21–23]; (2) quantum-state transfer from the squeezed light to atoms [24–28]; (3) nonlinear interactions among the individual spins with special systems, such as Bose-Einstein condensates [29–31], cold atoms in optical lattice [32], and atoms in the optical cavity [33–35]; (4) single [36] or multiple passes [37,38] of light through atoms. Compared to the former three methods, the last one does not require the projection by the measurement, nonclassical light field, and special systems, which would greatly simplify the experimental implementation; however, the obtainable degree of spin squeezing is limited by additional fluctuations imposed by the light field and/or the spherical nature of the spin distribution. Also, the present scheme is superior to that experimentally tested in a single-pass configuration for producing atomic spin squeezing between two excited states with a squeezed light field [24,25], where the lifetime of the atomic spin determined by the decay rates of the excited states is very short (about ~1 μs), whereas we examine the squeezing of atomic spin associated with the 5S ground-state hyperfine levels with long coherence decay time (~1 ms or even ~1 s [39,40]), which is vital for quantum memory.

II. THEORETICAL MODEL

The considered model, as shown in Fig. 1 (a), is similar to that adopted in Ref. [12]. Levels $|1\rangle$, $|2\rangle$, and $|3\rangle$ correspond, respectively, to the ground-state hyperfine levels $5S_{1/2}$ ($F = 2$), $5S_{1/2}$ ($F = 3$), and the excited state $5P_{1/2}$ in $D_1$ line of $^{85}$Rb atom with the ground-state hyperfine splitting of 3.036 GHz. The probe field $E_p$ (with frequency $\omega_p$ and Rabi frequency $\Omega_p$) and coupling field $E_c$ (with frequency $\omega_c$ and Rabi frequency $\Omega_c$) are relatively strong and tuned to resonance with the transitions $|1\rangle$-$|3\rangle$ and $|2\rangle$-$|3\rangle$, respectively. A third mixing field $E_m$ (with frequency $\omega_m$ and Rabi frequency $\Omega_m$), which can be generated from the coupling field with an acousto-optic modulator, off-resonantly couples levels $|1\rangle$ and $|3\rangle$. Obviously, one Stokes field $E_1$ can be created through the nondegenerate FWM process. In fact, as discussed in Ref. [12], the above generated FWM field can be equivalently viewed...
as scattering the mixing field off the atomic spin coherence $S$ preestablished by the strong (on-resonant) coupling and probe fields in the L-type EIT configuration formed by levels $|1\rangle$, $|2\rangle$, and $|3\rangle$, and the equivalent configuration is shown in Fig. 1(b).

III. RESULTS AND DISCUSSIONS

We now investigate the generation and manipulation of atomic spin squeezing via quantum coherence in this L-type atomic system, and employ the equivalent configuration in Fig. 1(b) to treat the generated Stokes field and atomic spin coherence. We assume that the Rabi frequency of the mixing field is far smaller than its frequency detuning, and the coupling and probe fields for creating the atomic spin coherence are as strong as scattering the mixing field off the atomic spin coherence $S$.

The exchange constant of motion $C_1$ [$C_1 = k_1 (a_1^\dagger S^+ + a_1 S)$], setting $\beta = \sqrt{C_1^2 - K_1^2}$, and solving the Heisenberg equations of motion, the solutions for the atomic spin operator $S$ and the Stokes field annihilation operator $a_1$ as functions of their initial values can be expressed as

$$S(t) = e^{iC_1t} \left[ \cos(\beta t) + \frac{iC_1}{\beta} \sin(\beta t) \right] S(0)$$
$$- \frac{i}{\beta} \sin(\beta t) a_1^\dagger(0) \right],$$

$$a_1(t) = e^{-iC_1t} \left[ \cos(\beta t) + \frac{iC_1}{\beta} \sin(\beta t) \right] a_1(0)$$
$$- \frac{i}{\beta} \sin(\beta t) S^+(0).$$

As seen in Fig. 1(b), if no laser fields are applied and we assume that all atoms are initially populated in level 1, then the mean values of the atomic spin components $S_x$, $S_y$, and $S_z$ are $\langle S_x \rangle = 0$, $\langle S_y \rangle = 0$, and $\langle S_z \rangle = \frac{1}{2}$, respectively, which means that the collective atomic spin is aligned along the $z$ axis. When only the coupling and probe fields are applied and the mixing field is not present, the collective atomic state is in a coherent superposition state $|\psi_0\rangle = (1/\sqrt{N_a}) \sum |i\rangle \left( \cos \theta |1\rangle - \sin \theta |2\rangle \right)$ with $1/2\theta = \Omega_c/\Omega_0$. In this case, the mean values of the atomic spin components $S_x$, $S_y$, and $S_z$ are $\langle S_x \rangle = -\frac{1}{2} \sin 2\theta$, $\langle S_y \rangle = 0$, and $\langle S_z \rangle = \frac{1}{2} \sin 2\theta$, respectively. So the new collective atomic spin (denoted by $S_2$) is now aligned along the $\theta, \varphi$ direction with $\theta = 2\theta$ and $\varphi = \pi$, where $S_2 = S \sin \theta \cos \varphi + S \cos \theta \sin \varphi + S \cos \theta \sin \varphi$. With the mean value being equal to 1/2, and the other two new atomic spin components $S_X$ and $S_Y$ in the plane orthogonal to the mean atomic spin direction are $S_X = S \sin \theta + S \cos \theta$ and $S_Y = S \theta$, having zero mean values. As a matter of fact, the new spin bases $S_X$, $S_Y$, and $S_Z$ just correspond to the atomic spin rotation by $\theta$ in the $(x, y)$ plane with respect to the original spin bases $S_x$, $S_y$, and $S_z$. This means that, by manipulating the quantum coherence between the lower doublet through changing the relative intensities of the coupling and probe fields, the direction of the atomic spin can be controlled. According to the criterion $\langle S_x^2 \rangle < |\langle S_z \rangle|/2$ or $\langle S_Z^2 \rangle < |\langle S_z \rangle|/2$ (i.e., the variance of the atomic spin component $S_x$ or $S_y$ being less than the absolute value of half the spin mean value) for verifying atomic spin squeezing [17–19], it is clear that since $\langle S_X^2 \rangle = |\langle S_x \rangle|/2 = 1/4$, no atomic spin squeezing can be created only with the coherent EIT fields. This is consistent with the previous statement [17–19] that atomic spin squeezing cannot be generated with two coherent laser fields in the L-type EIT configuration. As we make the assumption that the coupling and probe fields are strong that the established atomic spin coherence is strong enough to ensure that the mixing field has little influence on it, the mean value of the atomic spin does not change appreciably when the mixing field is applied. In this case, by using the expression for the atomic spin operator in Eq. (2) with the initial state of the atom-field system being $|\phi_0\rangle = (1/\sqrt{N_a}) \sum (\cos \theta |1\rangle - \sin \theta |2\rangle) |0\rangle$, $|1\rangle |0\rangle$ represents the $i$th atom in state $|1\rangle$ and the Stokes field

FIG. 1. (a) The double-Λ-type system of the $D_1$ transitions in the $^{85}$Rb atom coupled by the coupling ($E_c$), probe ($E_p$), and mixing ($E_m$) fields as defined on the experimental configuration used in Ref. [12], where $E_c$ and $E_p$ fields resonantly drive $|2\rangle-|3\rangle$ and $|1\rangle-|3\rangle$ transitions, respectively, and the corresponding Stokes field $E_p$ is generated through the FWM process. (b) The equivalent configuration of (a) with the two lower states driven by the atomic spin coherence $S$ induced by the strong on-resonant $E_c$ and $E_p$ fields in the Λ-type EIT configuration.
in vacuum), the variances of the spin components \( \langle \Delta S^x \rangle \) and \( \langle \Delta S^y \rangle \), and \( \langle S_Z \rangle / 2 \) can be expressed as

\[
\langle \Delta S^x \rangle = \frac{\cos^2\theta}{4} \left[ 1 + (2k_1^2/\beta^2)\sin^2\theta t \right]
+ \frac{\sin^2\theta}{4} \left[ k_1^2k_1^* - \cos^2\theta(A - k_1k_1^*)^2 \right],
\]
\[
\langle \Delta S^y \rangle = \frac{1 + (2k_1^2/\beta^2)\sin^2\theta t - B^2\sin^2\theta}{4},
\]
\[
\langle S_Z \rangle / 2 = \frac{\sin^2\theta + k_1k_1^* \cos^2\theta - (k_1k_1^* - 1)\cos\theta}{4},
\]

where \( k_{11} = e^{iC \cdot \theta} (\cos \beta t + iC \cdot \sin \beta t) \), \( A = (k_{11} + k_{11}^*)/2 \), and \( B = (k_{11} - k_{11}^*)/2i \). Obviously, when \( \theta = 0 \) or \( \theta = \pi \), \( \langle \Delta S^x \rangle = \langle \Delta S^y \rangle \equiv \langle S_Z \rangle / 2 \), and no atomic spin squeezing can be obtained in either the \( X \) or \( Y \) component. In what follows, we investigate the atomic spin squeezing as a function of the interaction time as well as the atomic spin coherence to see how the manipulation of atomic spin squeezing and creation of nearly perfect atomic squeezing can be realized via quantum coherence produced by the strong coupling and probe fields in the \( \Lambda \)-type EIT configuration.

Figures 2(a)–2(c) give the main results of this study, where the three-dimensional (3D) plots of the evolutions of \( \langle \Delta S^x \rangle \), \( \langle \Delta S^y \rangle \), and \( \langle S_Z \rangle / 2 \) as a function of the interaction time \( t \) and \( \theta \) with \( k_1 = 1 \) and \( C_1 = 30k_1 \) (with the same parameters as that in Ref. [12]) are depicted. Clearly, it can be seen that \( \langle \Delta S^x \rangle \), \( \langle \Delta S^y \rangle \), and \( \langle S_Z \rangle / 2 \), with the initial value of 1/4 at \( t = 0 \) and \( \theta = 0 \), exhibit oscillations as a function of the interaction time \( t \) and \( \theta \) with \( t \) periods of \( T \) (\( T = \pi/\beta \)), \( T/2 \), and \( T \), and \( \theta \) periods of \( \pi/2 \), \( \pi/4 \), and \( \pi \), respectively (where only one period of \( t \) and of \( \theta \) are displayed). By examining the sections through the 3D plots of \( \langle \Delta S^x \rangle \), \( \langle \Delta S^y \rangle \), and \( \langle S_Z \rangle / 2 \), either along the \( t \) or \( \theta \) axes, it is shown that when \( 0 < \theta < \pi/2 \), the variance \( \langle \Delta S^x \rangle \) becomes less than \( \langle S_Z \rangle / 2 \) as soon as the interaction begins, which indicates that the spin component \( S_y \) is squeezed; with the increase of the interaction time in the range of \( 0 < t < T/4 \), the degree of atomic spin squeezing is strengthened. When \( T/4 < t < T/2 \), whether there exists \( Y \)-component atomic spin squeezing or not would depend on the \( \theta \) value; with the increase of \( \theta \) in a moderate range near \( \pi/2 \), the degree and range of spin squeezing is increased, and

![FIG. 2. The 3D plots of the evolutions of \( \langle \Delta S^x \rangle \) (a), \( \langle \Delta S^y \rangle \) (b), and \( \langle S_Z \rangle / 2 \) (c) as a function of the interaction time (in terms of the normalized time \( k_1t \)) and the \( \theta \) value with \( k_1 = 1 \) and \( C_1 = 30k_1 \).](image-url)

when \( \theta = \pi/2 \) it can be realized over the whole interaction time range except at the points with \( t \) nearly equal to the integer times of \( T/2 \); a nearly perfectly squeezed atomic spin component can be achieved at \( t = T/4 \) and \( \theta = \pi/2 \). After \( t = T/2 \), the \( Y \)-component atomic spin squeezing exhibits an oscillation with the period of \( T/2 \). However, the \( X \)-component atomic spin squeezing displays a different behavior. It only exists in a limited range around \( t = T/2 \) within one interaction time period, and the degree of squeezing would be enhanced with the increase of \( \theta \) in the range of \( (0, \pi/4) \) and reduced in the range of \( (\pi/4, \pi/2) \); nearly perfect squeezing in the \( X \) component can be obtained in a very limited range around \( t = T/2 \) and \( \theta = \pi/4 \), whereas no spin squeezing would exist at \( \theta = \pi/4 \) and \( \pi/2 \). After that, the \( X \)-component of the atomic spin exhibits an oscillation as a function of interaction time with the period of \( T \).

Figures 3(a)–3(f) display the evolutions of \( \langle \Delta S^x \rangle \), \( \langle \Delta S^y \rangle \), and \( \langle S_Z \rangle / 2 \) as functions of \( \theta \) and the interaction time \( t \) with the cases of achieving nearly perfectly squeezed atomic spin in either the \( X \) or \( Y \) component. (a) \( \theta = \pi/2 \), (b) \( t = T/2 \), (c) \( \theta = 0.99\pi/4 \), (d) \( t = 0.99T/4 \), (e) \( \theta = 1.01\pi/4 \), and (f) \( t = 1.01T/4 \).
nearly perfect atomic spin squeezing achieved in either the X or Y component. It can be seen that the Y-component atomic spin squeezing can exist over the whole interaction time except at the points with \( t \) nearly equal to the integer times of \( T/2 \) when \( \theta = \pi/2 \) (i.e., the atomic spin is prepared in the maximum coherent state) [see Fig. 3(a)], and nearly perfectly squeezed atomic spin can be achieved at \( \theta = \pi/2 \) and \( t = T/4, 3T/4 \), whereas the X-component atomic spin squeezing can be obtained over the whole \( \theta \) values when \( t = T/2 \) except the points with \( \theta = \pi/4 \) and \( \theta = 3\pi/4 \) [see Fig. 3(b)]. However, the atomic spin squeezing in the X component only exists in a limited range of interaction time around \( t = T/2 \) when \( \theta \) is near \( \pi/4 \) [e.g., \( \theta = 0.99\pi/4 \) and \( \theta = 1.01\pi/4 \) in Figs. 3(c) and 3(e), respectively], whereas the Y-component atomic spin squeezing can be obtained over nearly the whole \( \pi \) range of \( \theta \) except \( \theta = 0 \) and \( \theta = \pi \) when \( t \) is a little smaller than \( T/4 \) [e.g., \( t = 0.99T/4 \) in Fig. 3(d)] and only in a limited range of \( \theta \) around \( \pi/2 \) when \( t \) is a little larger than \( T/4 \) [e.g., \( t = 1.01T/4 \) in Fig. 3(f)]. Therefore, by varying the \( \theta \) value (i.e., the relative intensities of the coupling and probe fields) or the interaction time, the generation and manipulation of atomic spin squeezing in either X or Y components can be conveniently realized.

As is well known, for the case of atomic spin squeezing, at any situation, if the variance of one spin component is squeezed, then the variance in the perpendicular direction would exhibit excess noise; that is, the Heisenberg uncertainty relationship should be satisfied. This is confirmed by comparing the product \( \langle \Delta S_X^2 \rangle \langle \Delta S_Y^2 \rangle \) to \( \langle S_x \rangle^2 \) as shown in Figs. 4(a)–4(f) for the corresponding situations in Figs. 3(a)–3(f). Clearly, for all cases, the Heisenberg uncertainty relationship \( \langle \Delta S_X^2 \rangle \langle \Delta S_Y^2 \rangle \geq \langle S_x \rangle^2 \) is always maintained. Furthermore, for the case of Figs. 3(a) and 3(b), the variances in the X and Y components almost satisfy the minimum-uncertainty relation, which means a nearly ideal squeezed atomic spin state can be established. Note that the Heisenberg uncertainty relationship in Figs. 4(a)–4(f) and the atomic spin squeezing in Figs. 3(a)–3(f) under various situations can be analytically confirmed as well by using Eqs. (4)–(6).

As pointed out in Refs. [17–19], generating atomic spin squeezing relies on the establishment of quantum-mechanical correlations among the elementary spins in the atomic ensemble. In the present case, the creation of atomic spin squeezing can be well understood in terms of the interaction between the laser fields and atomic medium [14,15]. As seen in Fig. 1(a), the Stokes field is produced through the FWM process, where every Stokes photon generation is accompanied by absorbing one mixing photon and one coupling photon and emitting one probe photon, and subsequent generation of atomic spin coherence excitation. The FWM process has proven to be an efficient way to produce the squeezed or entangled state for both atoms and fields with an atomic ensemble interacting with laser fields [5–15]. Equivalently, as shown in Fig. 1(b), the generated Stokes field can be regarded as the result of the frequency downconversion process through scattering the mixing field off the atomic spin coherence, which has the similar feature as the parametric downconversion process in a nonlinear optical crystal. Since the generation of a Stokes photon is always accompanied by an atomic spin-wave excitation, the downconverted frequency component (i.e., Stokes field) is strongly quantum correlated with the atomic spin wave. It can also be seen clearly from Eq. (1) that the effective Hamiltonian of the atom-field system has the same form as that for producing twin fields squeezing in the parametric downconversion process, and from Eqs. (2) and (3) that the atomic spin operator \( S(t) \) and Stokes field operator \( a_1(t) \) are both a linear combination of their initial values of \( S(0) \) and \( a_1(0) \), which indicates that the operators \( S(t) \) and \( a_1(t) \) are coupled together and strong correlation would exist between the Stokes field and atomic spin wave. Therefore, the generation of squeezing of the atomic spin as well as the Stokes field can be realized through the EIT-based FWM process. Moreover, as seen from Eqs. (4)–(6), for a finite interaction time, the variances of the spin components \( \langle \Delta S_X^2 \rangle \) and \( \langle \Delta S_Y^2 \rangle \), and \( \langle S_y \rangle \) all depend on the \( \theta \) value (i.e., the relative intensities of the coupling and probe fields determining the atomic spin coherence); therefore, by controlling the quantum coherence between the lower doublet, the manipulation of atomic spin squeezing in either X or Y components can be achieved.

It should be noted that in an atomic medium there will be realistic imperfections, such as finite coherence time and effect of Doppler broadening. Here, we did not take into account those effects, which can be justified by the following arguments. By using the configuration with laser beams propagating through the atomic medium in the same direction with small angles between them, as discussed in Ref. [12], the first-order Doppler broadening can be canceled in the double-A-type system, so the effect of Doppler broadening can be basically neglected. In addition, the interaction time for generating atomic spin squeezing depends on the strength of \( k_1 \). According to our previous experimental parameters [12], \( k_1 \) is of the order of several cm\(^{-1}\), and the estimated timescale for \( k_1t = 1 \) corresponds to about tens of picoseconds. This means that the interaction time for generating atomic spin squeezing is far shorter than the dephasing time of the atomic spin.

FIG. 4. The comparison of the product \( \langle \Delta S_X^2 \rangle \langle \Delta S_Y^2 \rangle \) (dashed red line) to \( \langle S_x \rangle^2 \) (solid dark line) for confirming the Heisenberg uncertainty relationship for the corresponding situations shown in Figs. 3(a)–3(f).
coherence (of the order of several microseconds for hot atoms, or even microseconds or seconds for cold atoms [39,40]). Therefore, the decoherence of the atomic spin coherence can be neglected as well. When decoherence and Doppler broadening are taken into account, no analytical expressions will be possible, and the discussions will be much more complicated. These issues will be analyzed and addressed in detail by using a Heisenberg-Langevin approach. Moreover, if the cold atoms are used, the realistic imperfections, such as decoherence times and Doppler broadening, would be very weak, and our results are well suited for such case.

IV. CONCLUSIONS

In conclusion, we have presented a convenient and efficient way to generate and manipulate atomic spin squeezing via quantum coherence established by the strong coupling and probe fields in the \( \Lambda \)-type EIT configuration. Moreover, only coherent input light fields are needed for controlling squeezing of the two quadrature components of the atomic spin and generating nearly perfect squeezing of either component, which holds great promise for realistic high-precision atomic-physics measurements (e.g., atomic clock), quantum coherent control, and quantum information processing.

ACKNOWLEDGMENTS

This work is supported by NBRPC (Grant No. 2012CB921804), National Natural Science Foundation of China (Grants No. 11574195, No. 11274225, No. 11321063, and No. 11404174), Key Basic Research Program of Shanghai Municipal Science and Technology Commission (Grant No. 14JC1402100), Shanghai Natural Science Foundation (Grant No. 14ZR145400), and the Open Research Fund Program of Jiangsu Provincial Key Lab of Center for Photon Manufacturing Science and Technology (Grant No. GZ201309).

[18] K. Hammerer, A. S. Sørensen, and E. S. Polzik, Rev. Mod. Phys. 82, 1041 (2010).